Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Core Mathematics 3R (6665_01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - └ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simp it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where}\ |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where}\ |pq| = |c|\ \text{and}\ |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	Factorise $4x^2-9=(2x-3)(2x+3)$ Use of common denominator $\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)-1(2x+3)+6}{(2x+3)(2x-3)}$ $=\frac{4x-6}{(2x+3)(2x-3)}$ $=\frac{2\cancel{(2x-3)}}{(2x+3)\cancel{(2x-3)}}=\frac{2}{2x+3}$	B1 M1 A1 A1 **(4)** **4 marks**
	Alternative where $4x^2-9$ is not factorised $\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)(4x^2-9)-1(2x+3)(4x^2-9)+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ $=\frac{2(2x-3)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ or $\frac{(4x-6)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ or $\frac{(2x-3)(8x^2-18)}{(2x+3)(2x-3)(4x^2-9)}$ $=\frac{(4x-6)\cancel{(4x^2-9)}}{(2x+3)(2x-3)\cancel{(4x^2-9)}}$ or $\frac{\overset{2}{\cancel{(4x-6)}}(4x^2-9)}{(2x+3)\cancel{(2x-3)}(4x^2-9)}$ $=\frac{2}{2x+3}$	M1 B1 A1 A1

B1 For **factorising** $4x^2-9$ to $(2x-3)(2x+3)$ at any point. Note that this is not scored for combining the terms $(2x-3)(2x+3)$ and writing the product as $4x^2-9$

M1 Use of common denominator – combines three fractions to form one. The denominator must be correct for their fractions and at least one numerator must have been adapted. Condone missing brackets.

$\frac{16x^3-24x^2-36x+54}{(4x^2-9)^2}$ is a correct intermediate stage but needs to be factorised and cancelled before A1

Examples of incorrect fractions scoring this mark are: $\frac{3(2x-3)-2x+3+6}{(2x+3)(2x-3)}$ missing bracket

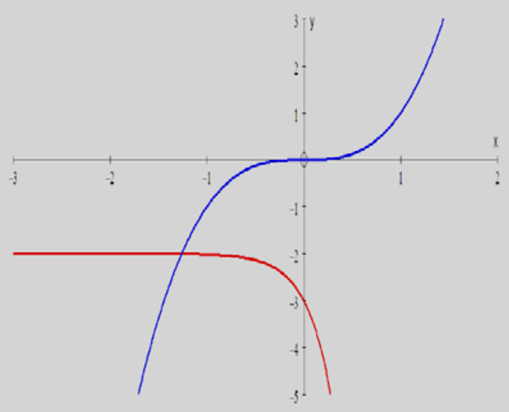
$\frac{3(4x^2-9)-4x^2-9+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ denominator correct and at least one numerator has been adapted.

A1 Correct simplified intermediate answer. It must be a CORRECT $\frac{\text{Linear}}{\text{Quadratic}}$ or $\frac{\text{Quadratic}}{\text{Cubic}}$

Accept versions of $\frac{4x-6}{(2x+3)(2x-3)}$ or $\frac{8x^2-18}{(2x+3)(4x^2-9)}$

A1 cao $=\frac{2}{2x+3}$

Allow recovery from invisible brackets for all 4 marks as the answer is not given.

Question Number	Scheme	Marks
2. (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\mathrm{e}^{4x} + 4x^3 + 8$	M1, A1
	Puts $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to give $x^3 = -2 - \mathrm{e}^{4x}$	A1 * (3)
(b)	 $y = x^3$	B1
	Shape of $y = -2 - \mathrm{e}^{4x}$	B1
	$y = -2 - \mathrm{e}^{4x}$ cuts y axis at (0,-3)	B1
	$y = -2 - \mathrm{e}^{4x}$ has asymptote at $y = -2$	B1 (4)
(c)	Only one crossing point	B1 (1)
(d)	–1.26376, -1.26126 Accept answers which round to these answers to 5dp	M1 A1 (2)
(e)	α = -1.26 and so turning point is at (-1.26, -2.55)	M1 A1cao (2) **12 marks**

(a)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$

A1* States or sets $\frac{dy}{dx} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - e^{4x}$.

(b)

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin. It must only appear in Quadrants 1 and 3 and have a gradient that is always $\geqslant 0$. The gradient should appear large at either end. Tolerate slips of the pen.See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - e^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - e^{4x}$ cutting or meeting the *y* axis at (0,-3). Its shape is not important.
Accept for the intention of (0,-3), -3 being marked on the y – axis as well as (-3,0)
Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - e^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as '-2' or indeed $x = -2$. See practice and qualification for acceptable solutions.

(c)

B1 Score for a statement to the effect that the graphs cross at one point. Accept minimal statements such as 'one intersection'. Do not award if their diagram shows more than one intersection. They must have a diagram (which may be incorrect)

(d)

M1 Awarded for applying the iteration formula once. Possible ways in which this can be scored are the sight of $\sqrt[3]{-2-e^{-4}}$, $\left(-2-e^{4\times-1}\right)^{\frac{1}{3}}$ or awrt -1.264

A1 Both values correct awrt –1.26376, -1.26126 5dps. The subscripts are unimportant for this mark. Score as the first and second values seen.

(e)

M1 Score for EITHER rounding their value in part (c) to 2 dp OR finding turning point of *C* by substituting a value of x generated from part (d) into $y = e^{4x} + x^4 + 8x + 5$ in order to find the *y* value. You may accept the appearance of a *y* value as evidence of finding the turning point (as long as an *x* value appears to be generated from part (d) and the correct equation is used.)

A1 (-1.26, -2.55) and correct solution only. It is a deduction and you cannot accept the appearance of a correct answer for two marks.

Question Number	Scheme	Marks
3. (i) (a)	$2\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}=\frac{5}{\sin x}$	B1
	Uses common denominator to give $2\sin^2 x-\cos^2 x=5\cos x$	M1
	Replaces $\sin^2 x$ by $(1-\cos^2 x)$ to give $2(1-\cos^2 x)-\cos^2 x=5\cos x$	M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, b = 5, c = -2)	A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give cos x =	M1
	cos x = $\frac{1}{3}$ only (rejects cos x = -2)	A1
	So x = 1.23 or 5.05	dM1A1 (4)
(ii)	Either Or	
	Either: $\tan\theta+\cot\theta\equiv\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$ Or: $\tan\theta+\cot\theta\equiv\tan\theta+\frac{1}{\tan\theta}$	B1
	Either: $\equiv\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ Or: $\equiv\frac{\tan^2\theta+1}{\tan\theta}$	M1
	Either: $\equiv\frac{2}{\sin 2\theta}$ Or: $\equiv\frac{1}{\cos^2\theta\times\frac{\sin\theta}{\cos\theta}}\equiv\frac{2}{\sin 2\theta}$	M1
	Either: $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$) Or: $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$)	A1 (4) **12 marks**
	Alternatives to Main Scheme	
3. (i) (a)	$2\tan x-\frac{1}{\tan x}=\frac{5}{\sin x}$ does not score any marks until	
	$\times\tan x\Rightarrow 2\tan^2 x+1=5\sec x$	B1, M1
	Replaces $\tan^2 x$ by $(\sec^2 x-1)$ to give $2(\sec x^2-1)+1=5\sec x$	M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, b = 5, c = -2)	A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give cos x = or $2\sec^2 x-5\sec x-3=0\Rightarrow\sec x=..$	M1
	cos x = $\frac{1}{3}$ only (rejects cos x = -2)	A1
	So x = 1.23 or 5.05	dM1A1 (4)
3. (ii)	$\tan\theta+\cot\theta=\lambda\text{cosec}2\theta\Rightarrow\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\lambda}{\sin 2\theta}=\left(\xcancel{\frac{\lambda}{2\sin\theta\cos\theta}}\right)$	B1
	$\times 2\sin\theta\cos\theta\Rightarrow 2\sin^2\theta+2\cos^2\theta=\lambda$	M1
	Factorises $2(\sin^2\theta+\cos^2\theta)=\lambda\Rightarrow 2=\lambda$	M1
	All above correct + a statement like ‘hence true’, ‘QED’	A1 (4)

(i)(a)

B1 Uses definitions $\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$ and $\text{cosec} x = \frac{1}{\sin x}$ to write the equation in terms of $\cos x$ and $\sin x$. Condone $5\text{cosec} x = \frac{1}{5\sin x}$ as the intention is clear.

Alternatively uses $\cot x = \frac{1}{\tan x}$ and $\text{cosec} x = \frac{1}{\sin x}$ to write the equation in terms of $\tan x$ and $\sin x$

This may be implied by later work that achieves $A\tan^2 x \pm B = C\sec x$

M1 Either uses common denominator and cross multiples, or multiplies each term by $\sin x \cos x$ to achieve an equation of the form equivalent to $A\sin^2 x \pm B\cos^2 x = C\cos x$. It may be seen on the numerator of a fraction

Alternatively multiplies by $\tan x$ to achieve $A\tan^2 x \pm B = C\sec x$

M1 Uses a correct Pythagorean relationship, usually $\sin^2 x = 1 - \cos^2 x$ to form a quadratic equation in terms of $\cos x$. In the alternative uses $\tan^2 x = \sec^2 x - 1$ to form a quadratic in sec *x*, followed by $\sec x = \frac{1}{\cos x}$ to form a quadratic equation in terms of $\cos x$

A1 Obtains $\pm K\left(3\cos^2 x + 5\cos x - 2\right) = 0$ (a =3, b = 5, c = -2)

(i)(b)

M1 Uses a standard method to solve their quadratic equation in $\cos x$ from **(i)(a) OR** sec*x* from an earlier line in (a)
See General Principles for Core Mathematics on how to solve quadratics

A1 $\cos x = \frac{1}{3}$ only Do not need to see –2 rejected

dM1 Uses arcos on their value to obtain at least one answer. It is dependent upon the previous M.
It may be implied by one correct answer

A1 Both values correct awrt 3sf x = 1.23 and 5.05.
Ignore any solutions outside the range. Any extra solutions in the range will score A0.
Answers in degrees will score A0.

(ii)

B1 Uses a definition of cot with matching expression for tan. Acceptable answers are

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$, $\frac{\sin\theta}{\cos\theta} + \frac{1}{\frac{\sin\theta}{\cos\theta}}$, $\tan\theta + \frac{1}{\tan\theta}$. Condone a miscopy on the sign. Eg Allow $\tan\theta - \frac{1}{\tan\theta}$

M1 Uses common denominator, writing their expression as a single fraction. In the examples given above, example 2 would need to be inverted. The denominator has to be correct and one of the terms must be adapted.

M1 Uses identities $\sin^2\theta + \cos^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\frac{\lambda}{\sin 2\theta}$

Alternatively uses $1 + \tan^2\theta = \sec^2\theta = \frac{1}{\cos^2\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\frac{\lambda}{\sin 2\theta}$. A line of $\frac{1}{\sin\theta\cos\theta}$ achieved on the lhs followed by $\lambda = \frac{1}{2}$ or 2 would imply this mark

A1 Achieves printed answer with no errors.

Allow for a different variable as long as it is used consistently.

Question Number	Scheme	Marks
4. (i)	$\frac{\mathrm{d}x}{\mathrm{d}y}=4\sec^2 2y\tan 2y$	B1
	Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$	M1
	Uses $\tan^2 2y=\sec^2 2y-1$ and $\sec 2y=\sqrt{x}$ to get $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of just x	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1*
		(4)
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=(x^2+x^3)\times\frac{2}{2x}+(2x+3x^2)\ln 2x$	M1 A1 A1
	When $x=\frac{\mathrm{e}}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3(\frac{\mathrm{e}}{2})+4(\frac{\mathrm{e}}{2})^2=3(\frac{\mathrm{e}}{2})+\mathrm{e}^2$	dM1 A1
		(5)
(iii)	$\mathrm{f}'(x)=\frac{(x+1)^{\frac{1}{3}}(-3\sin x)-3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{(x+1)^{\frac{2}{3}}}$	M1 A1
	$\mathrm{f}'(x)=\frac{-3(x+1)(\sin x)-\cos x}{(x+1)^{\frac{4}{3}}}$	A1
		(3)
		12 marks

(i)

B1 $\frac{dx}{dy} = 4\sec^2 2y \tan 2y$ or equivalent such as $\frac{dx}{dy} = 4\frac{\sin 2y \cos 2y}{\cos^4 2y}$

Accept $\frac{dx}{dy} = 2\sec 2y \tan 2y \times \sec 2y + 2\sec 2y \tan 2y \times \sec 2y$, $1 = 4\sec^2 2y \tan 2y \frac{dy}{dx}$

M1 Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to get an expression for $\frac{dy}{dx}$ in terms of y.

It may be scored following the award of the next M1 if $\frac{dx}{dy}$ has been written in terms of x.

Follow through on their expression but condone errors on the coefficient.

For example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2y \tan 2y}$ is OK as is $\frac{dy}{dx} = \frac{2}{\sec^2 2y \tan 2y}$

Do not accept y' s going to x' s. So for example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2x \tan 2x}$ is M0

M1 Uses $\tan^2 2y = \sec^2 2y - 1$ **and** $x = \sec^2 2y$ to get their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x

$\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2x\sqrt{(\sec^2 2y - 1)} = 2x\sqrt{x-1}$ is incorrect but scores M1

$\frac{dx}{dy} = 2\sec 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2\sec 2y\sqrt{(\sec^2 2y - 1)} = 2\sqrt{x}\sqrt{x-1}$ is incorrect but scores M1

The stating and use $1 + \tan^2 x = \sec^2 x$ is unlikely to score this mark.

Accept $1 + \tan^2 2y = \sec^2 2y \Rightarrow 1 + \tan^2 2y = x \Rightarrow \tan 2y = \sqrt{x-1}$. So $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan 2y} = \frac{1}{4x\sqrt{x-1}}$

Condone examples where the candidate adapts something to get the given answer

Eg. $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan^2 2y} = \frac{1}{4\sec^2 2y\left(\sec^2 2y - 1\right)} = \frac{1}{4x\sqrt{(x-1)}}$

A1* Completely correct solution. This is a ‘show that’ question and it is a requirement that all elements are seen.

(ii)

M1 Uses the product rule to differentiate $(x^2 + x^3)\ln 2x$. If the rule is stated it must be correct. It may be implied by their $u = .., u' = .., v = .., v' = ..$ followed by $vu' + uv'$. If the rule is neither stated nor implied only accept expressions of the form $\ln 2x \times (ax + bx^2) + (x^2 + x^3) \times \frac{C}{x}$

It is acceptable to multiply out the expression to get $x^2 \ln 2x + x^3 \ln 2x$ but the product rule must be applied to terms

A1 One term correct (unsimplified). Either $(x^2+x^3)\times\frac{2}{2x}$ or $(2x+3x^2)\ln 2x$

If they have multiplied out before differentiating the equivalent would be two of the four terms correct.

A1 A completely correct (unsimplified) expression $\frac{\mathrm{d}y}{\mathrm{d}x}=(x^2+x^3)\times\frac{2}{2x}+(2x+3x^2)\ln 2x$

dM1 Fully substitutes $x=\frac{e}{2}$ (dependent on previous M mark) into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}=....$ Implied by awrt 11.5

A1 $\frac{\mathrm{d}y}{\mathrm{d}x}=3(\frac{\mathrm{e}}{2})+\mathrm{e}^2$ Accept equivalent simplified forms such as $\frac{\mathrm{d}y}{\mathrm{d}x}=1.5\mathrm{e}+\mathrm{e}^2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}(1.5+\mathrm{e})$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}(2\mathrm{e}+3)}{2}$

(iii)

M1 Uses quotient rule with $u=3\cos x$, $v=(x+1)^{\frac{1}{3}}$, $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{2}{3}}$.

If the rule is quoted it must be correct. It may be implied by their $u=3\cos x$, $v=(x+1)^{\frac{1}{3}}$, $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{2}{3}}$ followed by $\frac{vu'-uv'}{v^2}$

Additionally this could be scored by using the product rule with $u=3\cos x$, $v=(x+1)^{-\frac{1}{3}}$ $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{4}{3}}$. If the rule is quoted it must be correct. It may be implied by their $u=3\cos x$, $v=(x+1)^{-\frac{1}{3}}$ $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{4}{3}}$ followed by $vu'+uv'$

If it is not quoted nor implied only accept either of the two expressions

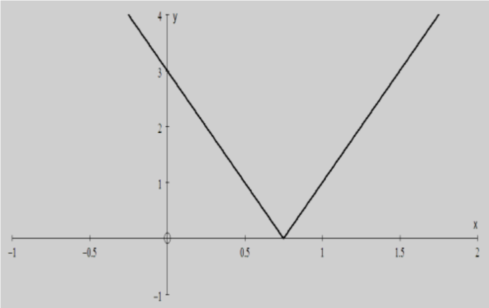
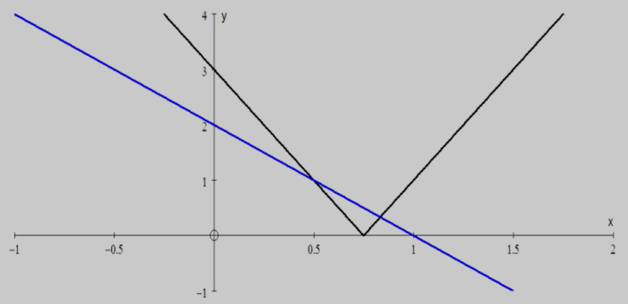
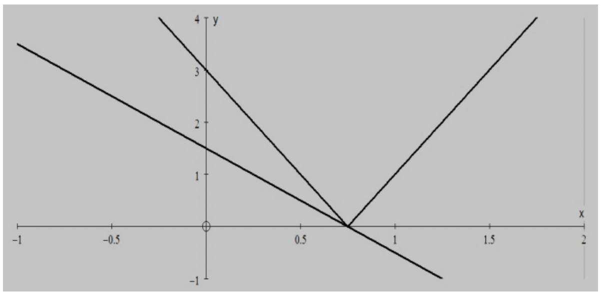
1) Using quotient form $\frac{(x+1)^{\frac{1}{3}}\times\pm A\sin x-3\cos x\times B(x+1)^{-\frac{2}{3}}}{\left((x+1)^{\frac{1}{3}}\right)^2}$ or $\frac{(x+1)^{\frac{1}{3}}\times\pm A\sin x-3\cos x\times B(x+1)^{-\frac{2}{3}}}{(x+1)^{\frac{1}{9}}}$

2) Using product form $(x+1)^{-\frac{1}{3}}\times\pm A\sin x+3\cos x\times B(x+1)^{-\frac{4}{3}}$

A1 A correct gradient. Accept $\mathrm{f}'(x)=\frac{(x+1)^{\frac{1}{3}}(-3\sin x)-3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{\left((x+1)^{\frac{1}{3}}\right)^2}$

or $\mathrm{f}'(x)=(x+1)^{-\frac{1}{3}}\times-3\sin x+3\cos x\times-\frac{1}{3}(x+1)^{-\frac{4}{3}}$

A1 $\mathrm{f}'(x)=\frac{-3(x+1)(\sin x)-\cos x}{(x+1)^{\frac{4}{3}}}$ oe. or a statement that $\mathrm{g}(x)=-3(x+1)(\sin x)-\cos x$ oe.

Question Number	Scheme	Marks
5. (a)	 V shaped graph	B1
	Touches x axis at ¾ and cuts y axis at 3	B1
		(2)
(b)	 Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x	M1
	Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ or $x>\frac{5}{6}$ or $x<\frac{1}{2}$	A1
	$x<\frac{1}{2}$ or $x>\frac{5}{6}$	dM1A1
		(4)
(c)	 Draws graph Or solves $\lvert 4x-3\rvert=1\tfrac{1}{2}-2x$ to give one soln x = ¾	M1
	Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\mathbb{R},)$ $x\neq\frac{3}{4}$, or $x<\frac{3}{4}$, $x>\frac{3}{4}$	A1
		(2)
		(8 marks)

(a)

B1 A ‘V’ shaped graph. The position is not important. Do not accept curves. See practice and qualification items for clarity. Accept a V shape with a ‘dotted’ extension of $y = 4x - 3$ appearing under the *x* axis.

B1 The graph **meets** the x axis at $x = \frac{3}{4}$ and **crosses** the y axis at $y = 3$. Do not allow multiple meets or crosses

If they have lost the previous B1 mark for an extra section of graph underneath the *x* axis allow for **crossing** the x axis at $x = \frac{3}{4}$ and **crosses** the y axis at $y = 3$.

Accept marked elsewhere on the page with *A* and *B* marked on the graph and $A = \left(\frac{3}{4}, 0\right)$ and $B = (0, 3)$

Condone $\left(0, \frac{3}{4}\right)$ and (3, 0) marked on the correct axis

(b)

M1 Attempts to solve $|4x-3| ... 2-2x$ finding at least one solution. You may see … replaced by either = or >

Accept as evidence $\pm 4x \pm 3 = 2 - 2x \Rightarrow x = ..$

Accept as evidence $\pm 4x \pm 3 > 2 - 2x \Rightarrow x > ..$, or $x < ..$

A1 Both critical values $x = \frac{5}{6}$ and $x = \frac{1}{2}$, or one inequality, accept $x > \frac{5}{6}$ or $x < \frac{1}{2}$

Accept $x = 0.83$ and $x = 0.5$ for the critical values

Accept both of these answers with no incorrect working for both marks

dM1 Dependent upon the previous M, this is scored for selecting the outside region of their two points.

Eg if M1 has been scored for $4x - 3 = 2 - 2x \Rightarrow x = 0.83$ and $-4x - 3 = 2 - 2x \Rightarrow x = -2.5$

A correct application of M1 would be $x < -2.5, x > 0.83$

A1 Correct answer only $x < \frac{1}{2}$ or $x > \frac{5}{6}$.

Accept $x < 0.5, x > 0.8\dot{3}$

(c)

M1 **Either** sketch both lines showing a single intersection at the point $x = \frac{3}{4}$

Or solves $|4x-3| = 1\tfrac{1}{2} - 2x$ using both $4x - 3 = 1\tfrac{1}{2} - 2x$ and $-4x + 3 = 1\tfrac{1}{2} - 2x$ **giving one solution** $x = \frac{3}{4}$

Accept $|4x-3| > 1\tfrac{1}{2} - 2x$ using both $4x - 3 > 1\tfrac{1}{2} - 2x$ and $-4x + 3 > 1\tfrac{1}{2} - 2x$ **giving one solution** $x ... \frac{3}{4}$

If two values are obtained using either method it is M0A0

A1 States that the solution set is all values apart from $x = \frac{3}{4}$. Do not isw in this question. Score their final statement. Accept versions of all values of *x* except $x = \frac{3}{4}$ or $x \in \mathbb{R},\ x \neq \frac{3}{4}$, or $x < \frac{3}{4},\ x > \frac{3}{4}$

Question Number	Scheme	Marks
6.(a)	$\mathrm{f}(x) > k^2$	B1
		(1)
(b)	$y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow \mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$	A1
		(3)
(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = \mathrm{e}^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{\mathrm{e}}{\sqrt[6]{8}}\right) = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	A1
		(4)
(d)	$\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2$	M1
	$\Rightarrow \mathrm{fg}(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
(e)	$\mathrm{fg}(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2}$ **only**	A1
		(2) **12 marks**
(alt c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 2 + \ln x + \ln 2 + 2\ln x + \ln 2 + 3\ln x = 6$	
	$\Rightarrow 3\ln 2 + 6\ln x = 6$	
	$\Rightarrow \ln x = 1 - \frac{1}{2}\ln 2$	M1
	$\Rightarrow x = e^{1-\frac{1}{2}\ln 2}, = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	M1, A1
(alt e)		(4)
	$\Rightarrow 2\ln(2x) = \ln(2k^2 - k^2)$	
	$\Rightarrow \ln(2x)^2 = \ln(k^2), \Rightarrow 4x^2 = k^2 \Rightarrow x = \frac{k}{2}$	M1, A1

(a)

B1 States the correct range for f. Accept $\mathrm{f}(x) > k^2, \mathrm{f} > k^2, Range > k^2, \left(k^2, \infty\right), y > k^2$ Range is greater than k

Do not accept $\mathrm{f}(x) \geq k^2, x > k^2, [k^2, \infty)$

(b)

M1 Attempts to make x or a swapped y the subject of the formula. Score for $y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y \pm k^2$ and proceeding to $x = \ln\ldots$. The minimum expectation is that e^{2x} is made the subject before taking ln's

dM1 Dependent upon the previous M having been scored. It is for proceeding by firstly taking ln's of the whole rhs, not the individual elements, and then dividing by 2. Score M1, dM1 for writing down

$x = \frac{1}{2}\ln(y \pm k^2)$ or alternatively $y = \frac{1}{2}\ln(x \pm k^2)$. Condone missing brackets for this mark.

A1 The correct answer in terms of x including the bracket **and** the domain $\mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$.

Accept equivalent answers like $y = 0.5\ln\left|x - k^2\right|$, Domain greater than k^2, $\left(k^2, \infty\right)$

(c)

M1 Attempts to solve equation by writing down $\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$

M1 Uses addition laws of logs to write in the form $\ln Ax^n = 6$

M1 Takes exp's (correctly) and proceeds to a solution for $x = ..$

A1 Correct solution and correct answer. $x = \frac{\mathrm{e}}{\sqrt{2}}$. You may ignore any reference to $x = -\frac{\mathrm{e}}{\sqrt{2}}$

Special caseS. Candidate who solve (and treat it as though it was bracketed)

S. Case 1 $\ln 2x + \ln 2x^2 + \ln 2x^3 = 6 \Rightarrow \ln 2x + 2\ln 2x + 3\ln 2x = 6 \Rightarrow 6\ln 2x = 6 \Rightarrow x = \frac{\mathrm{e}}{2}$

S. Case 2 $\ln 2x + \ln(2x)^2 + \ln(2x)^3 = 6 \Rightarrow 6\ln 2x = 6 \Rightarrow \ln 2x = 1 \Rightarrow x = \frac{\mathrm{e}}{2}$

S. Case 3 $\ln 2x + \ln(2x)^2 + \ln(2x)^3 = 6 \Rightarrow \ln 2x + \ln 4x^2 + \ln 8x^3 = 6 \Rightarrow \ln 64x^6 = 6 \Rightarrow 64x^6 = \mathrm{e}^6 \Rightarrow x = \frac{\mathrm{e}}{2}$

scores M0 (Incorrect statement/ may be implied by subsequent work), M1 (Correct ln laws), M1 (Correct method of arriving at x=) , A0

(d) For the purposes of marking you can score (d) and (e) together

M1 Correct order of applying g before f to give a correct unsimplified answer. Accept y =

Accept versions of $\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2, y = e^{\ln(2x)^2} + k^2$

A1 A correct simplified answer $\mathrm{fg}(x) = (2x)^2 + k^2$, or $\mathrm{fg}(x) = 4x^2 + k^2$. Accept y =

(e)

M1 Sets the answer to (d) in the form $Ax^2 + k^2 = 2k^2$, where $A = 2$ or 4 and proceeds in the correct order to reach an equation of the form $Ax^2 = k^2$.

In the alternative method it would be for reaching $\ln(Ax^2) = \ln(k^2)$, $A = 2$ or 4 or any equivalent form ln…= ln…

A1 $x = \frac{k}{2}$ **only.** The answer $x = \pm\frac{k}{2}$ is A0.

Question Number	Scheme	Marks
7.(a)	$R=\sqrt{\left(6^2+2.5^2\right)}=6.5$	B1
	$\tan\alpha=\frac{2.5}{6}, \quad \Rightarrow \quad \alpha=\text{awrt }0.395$	M1A1
		(3)
(b)	$(0,6)$,	B1
	awrt $(1.97,0) \quad (5.11,0)$	M1A1
		(3)
(c)	$H\text{max}=18.5, H\text{min}=5.5$	M1A1A1
		(3)
(d)	Sub $H=16$ and proceed to $'6.5'\cos\left(\frac{2\pi t}{52}\pm'0.395'\right)=4$	M1
	$\left(\frac{2\pi t}{52}-'0.395'\right)=\text{awrt }0.91$	A1
	$t=\left(awrt\,0.908\pm'0.395'\right)\times\frac{52}{2\pi}=11\ (10.78)$	dM1A1
	$\left(\frac{2\pi t}{52}\pm'0.395'\right)=awrt\,2\pi-0.908\Rightarrow t=48\,(47.75)$	ddM1A1
		(6) **(15 marks)**

(a)

B1 $R=6.50, \frac{13}{2}$. Accept $R=$ awrt 6.50. Do not accept $R=\pm 6.50$

M1 For reaching $\tan\alpha=\pm\frac{2.5}{6}$ *or* $\tan\alpha=\pm\frac{6}{2.5}$.

If R has been attempted first then only accept $\sin\alpha=\pm\frac{2.5}{'R'}$ *or* $\cos\alpha=\pm\frac{6}{'R'}$

A1 Correct value $\alpha=awrt\ 0.395$. The answer in degrees $22.6°$ is A0

(b)

B1 The correct y intercept. Accept $y=6, (0,6)$, awrt y = 6.00, $f(0)=6$ or it marked on the curve.

Do not accept $(6,0)$

M1 Attempt to find either x intercept from $\frac{\pi}{2}+\text{their }0.395$, or $\frac{3\pi}{2}+\text{their }0.395$

If the candidate is working in degrees accept $90+\text{their }22.6$ or $270+\text{their }22.6$

One answer correct will imply this.

A1 Both answers correct. Accept awrt $(1.97,0)$ and $(5.11,0)$, Accept $x=1.97$ and $x=5.11$ or both being marked on the curve. Do not accept $(0,1.97)$ and $(0,5.11)$ for both marks

In degrees accept $(112.6,0)$ and $(292.6,0)$

(c)

M1 Attempts either $12+'R'$ OR $12-'R'$

A1 Either of 18.5 or 5.5. Accept one of these for two marks

A1 Both 18.5 and 5.5.

Accept for 3 marks answers just written down with limited or no working.

Attempted answers via differentiation will be few and far between but can score 3 marks.

M1 Differentiates to $H' = \pm A\sin\left(\frac{2\pi t}{52}\right) \pm B\cos\left(\frac{2\pi t}{52}\right)$, followed by $H' = 0$

$\Rightarrow \tan\left(\frac{2\pi t}{52}\right) = \left(\frac{5}{12}\right) \Rightarrow t = \text{awrt } 3.2.., 29.2...$

For the M to be scored they need to sub one value of t (which may not be correct) into $H =$

A1 Either of 18.5 or 5.5. A 1 Both 18.5 and 5.5.

(d)

M1 Substitutes $H = 16$ into the equation for H and proceeds to $'6.5'\cos\left(\frac{2\pi t}{52} \pm '0.395'\right) = 4$

Accept for this mark $'6.5'\cos\left(x \pm '0.395'\right) = 4$

A1 A correct intermediate line, which may be implied by a correct final answer. Follow through on their numerical value of α

Accept in terms of '*t*' $\left(\frac{2\pi t}{52} - '0.395'\right) = awrt\, 0.91$ or in terms of '*x*' $\left(x - '0.395'\right) = awrt\, 0.91$

Accept in terms of '*t*' $\left(\frac{2\pi t}{52} - '0.395'\right) = \text{invcos}\,\frac{4}{6.5}$

dM1 A full method to find one value of t. It is dependent upon the previous M mark having been awarded.

Accept $t = \left(their\, 0.908 \pm '0.395'\right) \times \frac{52}{2\pi}$.

Don't be overly concerned with the mechanics of this but the '0.395' the 2π and the 52 must have been used to find *t*.

A1 One correct value of *t* with a correct solution. Both M's must have been scored.

Accept awrt 10.7/10.8 or 11 or 47.7/47.8 or 48.

ddM1 A full method to find a secondary value of t. It is dependent upon both previous M's.

$$\left(\frac{2\pi t}{52} \pm their\,'0.395'\right) = awrt\, 2\pi - their\, 0.91 \Rightarrow t = ..$$

Don't be overly concerned with the mechanics of this but the '0.395' the 2π and the 52 must have been used to find *t*.

A1 Accept 11 and 48 coming from awrt 10.8/10.7 and 47.7/47.8. Both values of *t* need to be correct and have been rounded from *t* values that were correct to 1 dp. The intermediate values can be implied by seeing the whole calculation as written out in the mark scheme

Answers obtained by graphical or numerical means are not acceptable.

Answers obtained from degrees are perfectly acceptable only if degrees were used throughout (d) with π, being replaced by 180° in the formula and the answers in degrees converted back to radians at the end.

Mixed units can only score the first M1A1

$$6.5\cos\left(\frac{2\pi t}{52} - '22.6'\right) = 4 \Rightarrow \left(\frac{2\pi t}{52} - '22.6'\right) = awrt\, 52.0$$

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